

Analyzing the Impact of Events Through Surveys

Online Appendix

Andrew Bertoli
IE University

Laura Jakli
Harvard Business School

Henry Pascoe
IE University

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1 Pre-event/Post-event Survey Designs in APSR, AJPS, and JOP (2015-24)

Table 1 lists more than 50 studies that we identified as using the pre-event/post-event survey design. As the table shows, these studies examine a wide range of important events and outcomes of great interest to political scientists. Muñoz, Falcó-Gimeno, and Hernández (2020: A7) also provide a table with information on 44 studies published in a variety of journals that focus specifically on unexpected events. Their list spans the fields of political science, sociology, and economics. In short, studying the impact of important events through surveys stands out as a key part of social science, and it will likely remain common well into the future.

Table 1: Selected pre-event/post-event survey design studies

Study	Event(s)	Outcome(s)
Balcells, Tellez, and Villamil 2024	Russian invasion of Ukraine	Spanish nationalism
Bartels, Horowitz, and Kramon 2023	Kenyan supreme court elections	Judicial support; partisan backlash
Cohen et al. 2023	Bolsonaro election	Allegiance to political system
Epifanio, Giani, and Ivandic 2023	2005 London bombings	Support toward curbing freedoms
Harding and Nwokolo 2023	Boko Haram attack	Political trust; national identification; ethnic identification
Leininger et al. 2023	Brief loss of the right to vote	External efficacy
Mettler, Jacobs, and Zhu 2023	Republicans gaining control of Congress	Support for the Affordable Care Act
Pop-Eleches and Way 2023	Repression of Moldova electoral protests	Opposition support
Sexton and Zürcher 2023	German aid program	Economic perceptions; attitudes toward government and insurgency
Singh and Tir 2023	Terrorist attacks	Reported electoral participation
Yang 2023	Corporate headquarters relocation	Gubernatorial approval
Bateson and Weintraub 2022	2016 US presidential election	Trust in the United States
Berliner and Wehner 2022	Audits	Approval of Mayors
Bisbee and Honig 2022	COVID outbreak	Support for mainstream candidates in the US and France
Bove, Salvatore, and Elia 2022	Peacekeeper deployment	Perceived security and well-being
Breton and Eady 2022	2015 Paris terrorist attacks	Attitudes toward Syrian refugees
Hale 2022	Invasion of Crimea	Putin's popularity in Russia
Holman, Merolla, and Zechmeister 2022	2017 Manchester terrorist attack	Support for Teresa May
Kalla and Broockman 2022	Personal persuasion campaigns	Affective polarization
Kaslovsky 2022	Visits by senators to particular locations	Support for senators

Table 1: Selected pre-event/post-event survey design studies (continued)

Study	Event(s)	Outcome(s)
Ayoub, Page, and Whitt 2021	Pride event	Attitudes toward LGBT+ community
Croke 2021	Anti-malaria campaign	Leader approval
Goldsmith, Horiuchi, and Matush 2021	High-level state visits	Approval of visiting leader
Heide-Jørgensen 2021	Elections in Denmark	Coherence of welfare attitudes
Ketchley and El-Rayyes 2021	Protests	Perceptions of democracy
Reny and Newman 2021	George Floyd protests	Attitudes toward the police and African-Americans
Sances and Clinton 2021	Trump election	Opinions toward Affordable Care Act
Slothuus and Bisgaard 2021	Party position reversal	Policy approval
Stauffer 2021	2018 midterm elections	Symbolic benefit of representative government
Bartels and Kramon 2020	Presidential transitions in Ghana	Public support for judicial power
Batto and Beaulieu 2020	Legislative brawl	Evaluation of the legislature
Brierley, Kramon, and Ofosu 2020	Ghana parliamentary debates	Candidate evaluation
Lehmann and Masterson 2020	Cash transfers to Syrian refugees	Resentment toward refugees
Mikulaschek, Pant, and Tesfaye 2020	Iraqi PM resignation	Support for violent opposition; public service provision optimism
Schuster 2020	Campaign spending	Candidate support
Enos, Kaufman, and Sands 2019	Los Angeles riot	Support for public schools
Frye and Borisova 2019	Election; protest	Trust in government
Hobbs and Lajevardi 2019	Trump campaign and election events	Muslim Americans' reported public space avoidance
Singh and Thornton 2019	Elections	Partisanship
Alkon and Wang 2018	Pollution reduction intervention	Regime evaluation
Bisgaard and Slothuus 2018	Party cue	Perceptions of public deficit
Flesken 2018	Romanian electoral campaign	National and ethnic salience
Baker et al. 2016	Party brand change	Party support/identification
Bishin et al. 2016	Supreme court ruling	Attitudes toward gays and lesbians
Holbein and Hillygus 2016	Voter preregistration	Voter turnout
Bisgaard 2015	Economic shock	Attributions of responsibility
Branton et al. 2015	2006 immigration protests	Immigration policy preferences
Christenson and Glick 2015	Affordable Care Act challenge	Opinion of the Supreme Court
Peffley, Hutchison, and Shamir 2015	Political tolerance	Terrorist attacks
Tesler 2015	Elite political communication	Public opinion
Hirano et al. 2015	Primary campaigns and elections	Perceptions of candidates

2 External Validity

Consider the context where the target parameter that we want to estimate is the average causal effect for the population of interest:

$$\bar{\tau}_k = \frac{1}{N} \sum_{i=1}^N (y_{ikt} - y_{ikc})$$

Like when we estimated the average treatment effect for the Wave 2 respondents in our baseline model, the estimator that we will use to estimate $\bar{\tau}_k$ is the average difference between the Wave 2 and Wave 1 respondents' answers to question k of the survey:

$$\hat{\tau}_k = \hat{\tau}_{k|r_a=1} = \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikto|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikbo|r_b=1}$$

The bias in $\hat{\tau}_k$ can then be written as

$$Bias(\hat{\tau}_k) = Bias_{\mathbf{X}}(\hat{\tau}_k) + Bias_{\mathbf{T}}(\hat{\tau}_k) + Bias_{\mathbf{A}}(\hat{\tau}_k) + Bias_{\mathbf{M}}(\hat{\tau}_k) + Bias_{\mathbf{H}}(\hat{\tau}_k) \quad (\text{A1})$$

where

$$Bias_{\mathbf{X}}(\hat{\tau}_k) = \bar{y}_{ikb|r_a=1} - \bar{y}_{ikb|r_b=1}$$

$$Bias_{\mathbf{T}}(\hat{\tau}_k) = \bar{y}_{ikc|r_a=1} - \bar{y}_{ikbc|r_a=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_k) = \bar{y}_{ikbc|r_a=1} - \bar{y}_{ikb|r_a=1}$$

$$Bias_{\mathbf{M}}(\hat{\tau}_k) = \bar{\epsilon}_{kt|r_a=1} - \bar{\epsilon}_{kb|r_b=1}$$

$$Bias_{\mathbf{H}}(\hat{\tau}_k) = \bar{\tau}_{k|r_a=1} - \bar{\tau}_k$$

This expression for the overall bias is the same as in our baseline model, except for the $Bias_{\mathbf{H}}(\hat{\tau}_k)$ term that accounts for potential bias caused by the Wave 2 respondents having a heterogeneous treatment effect compared to the treatment effect in the overall population.

Deriving the bias in $\hat{\tau}_k$ is trivial. Begin by noting that $Bias(\hat{\tau}_{k|r_a=1}) = E[\hat{\tau}_{k|r_a=1}] - \bar{\tau}_{k|r_a=1}$, which can be rewritten as $E[\hat{\tau}_{k|r_a=1}] = Bias(\hat{\tau}_{k|r_a=1}) + \bar{\tau}_{k|r_a=1}$. Since $\hat{\tau}_k = \hat{\tau}_{k|r_a=1}$, we have $E[\hat{\tau}_k] = E[\hat{\tau}_{k|r_a=1}]$, so we can change the expression to $E[\hat{\tau}_k] = Bias(\hat{\tau}_{k|r_a=1}) + \bar{\tau}_{k|r_a=1}$. The bias in $\hat{\tau}_k$ is then just

$$\begin{aligned}
Bias(\hat{\tau}_k) &= E[\hat{\tau}_k] - \bar{\tau}_k \\
&= Bias(\hat{\tau}_{k|r_a=1}) + \bar{\tau}_{k|r_a=1} - \bar{\tau}_k \\
&= Bias(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{H}}(\hat{\tau}_k) \\
&= Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{H}}(\hat{\tau}_k) \\
&= Bias_{\mathbf{X}}(\hat{\tau}_k) + Bias_{\mathbf{T}}(\hat{\tau}_k) + Bias_{\mathbf{A}}(\hat{\tau}_k) + Bias_{\mathbf{M}}(\hat{\tau}_k) + Bias_{\mathbf{H}}(\hat{\tau}_k)
\end{aligned}$$

where

$$\begin{aligned}
Bias_{\mathbf{X}}(\hat{\tau}_k) &= Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1}) = \bar{y}_{ikb|r_a=1} - \bar{y}_{ikb|r_b=1} \\
Bias_{\mathbf{T}}(\hat{\tau}_k) &= Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1}) = \bar{y}_{ikc|r_a=1} - \bar{y}_{ikbc|r_a=1} \\
Bias_{\mathbf{A}}(\hat{\tau}_k) &= Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1}) = \bar{y}_{ikbc|r_a=1} - \bar{y}_{ikb|r_a=1} \\
Bias_{\mathbf{M}}(\hat{\tau}_k) &= Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1}) = \bar{e}_{kt|r_a=1} - \bar{e}_{kb|r_b=1} \\
Bias_{\mathbf{H}}(\hat{\tau}_k) &= \bar{\tau}_{k|r_a=1} - \bar{\tau}_k
\end{aligned}$$

3 Quota Sampling

In this design, we survey two groups of people before and after the event, selecting participants based on covariates to try to make the two groups similar to each other and to the total population. Let n be the total number of people who we consider surveying, with n_a denoting the number in Wave 2 and n_b denoting the number in Wave 1. As in the paper, let $r_{ia} \in \{0, 1\}$ denote whether individual i completed the survey in Wave 2, and let $r_{ib} \in \{0, 1\}$ denote whether individual i completed the survey in Wave 1. In addition, let $q_i \in \{0, 1\}$ denote whether individual i is in our quota group for either Wave 1 or Wave 2.

Building off this notation, we can let g_a denote the number of people in the Wave 2 quota group and g_b denote the number of people in the Wave 1 quota group. We can also define $g = g_a + g_b$ as the total number of people in our quota sample. Individuals were not randomized to be contacted in Wave 1 or Wave 2, so g, g_a, g_b are all parameters, not random variables.

3.1 Proof of Proposition 1

Proposition 1. *Bias in the quota sampling design is given by*

$$Bias(\hat{\tau}_{k|r_a=1,q=1}) = Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1,q=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1,q=1})$$

where

$$Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1,q=1}) = \bar{y}_{ikb|r_a=1,q=1} - \bar{y}_{ikb|r_b=1,q=1}$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=1}) = \bar{y}_{ikc|r_a=1,q=1} - \bar{y}_{ikbc|r_a=1,q=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=1}) = \bar{y}_{ikbc|r_a=1,q=1} - \bar{y}_{ikb|r_a=1,q=1}$$

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1,q=1}) = \bar{\epsilon}_{kt|r_a=1,q=1} - \bar{\epsilon}_{kb|r_b=1,q=1}$$

Proof. The causal parameter that we want to estimate is the average causal effect of the event on the Wave 2 quota group's truthful responses to question k of the survey:

$$\bar{\tau}_{k|r_a=1,q=1} = \frac{1}{g_a} \sum_{i=1}^{g_a} (y_{ikt|r_a=1,q=1} - y_{ikc|r_a=1,q=1})$$

The statistic that we will use to estimate this parameter is the average difference between the reported answers of the g_a respondents who completed our survey in Wave 2 and the g_b respondents who completed it in Wave 1.

$$\hat{\tau}_{k,r_a=1,q=1} = \frac{1}{g_a} \sum_{i=1}^{g_a} y_{ikto|r_a=1,q=1} - \frac{1}{g_b} \sum_{i=1}^{g_b} y_{ikbo|r_b=1,q=1}$$

Following the same procedures from the analysis in Section 2 of the paper, the bias in this estimator can be rewritten as

$$Bias(\hat{\tau}_{k|r_a=1,q=1}) = Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1,q=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1,q=1})$$

where

$$\begin{aligned}
Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1,q=1}) &= \bar{y}_{ikb|r_a=1,q=1} - \bar{y}_{ikb|r_b=1,q=1} \\
Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=1}) &= \bar{y}_{ikc|r_a=1,q=1} - \bar{y}_{ikbc|r_a=1,q=1} \\
Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=1}) &= \bar{y}_{ikbc|r_a=1,q=1} - \bar{y}_{ikb|r_a=1,q=1} \\
Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1,q=1}) &= \bar{\epsilon}_{kt|r_a=1,q=1} - \bar{\epsilon}_{kb|r_b=1,q=1}
\end{aligned}$$

□

The difference between this overall bias term and the $Bias(\hat{\tau}_{k|r_a=1})$ expression that we derived in Section 2 of the paper is that this term restricts the focus to our quota sample.

3.2 Proof of Proposition 2

Proposition 2. *Quota designs reduce bias if and only if*

$$\begin{aligned}
\left| Bias(\hat{\tau}_{k|r_a=1,q=1}) \right| < & \left| \left(\frac{g_a}{n_a} \right) Bias(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a} \right) Bias(\hat{\tau}_{k|r_a=1,q=0}) + \right. \\
& \left. \left(\frac{g_a}{n_a} - \frac{g_b}{n_b} \right) (\bar{y}_{ikbo|r_b=1,q=1} - \bar{y}_{ikbo|r_b=1,q=0}) \right|
\end{aligned}$$

When the inequality is flipped, quota sampling **amplifies** bias.

Proof. Whether quota sampling improves on our baseline model depends in part on whether the bias in the excluded group is smaller or in the opposite direction as the bias in the quota group. It also depends, to some extent, on external validity considerations, since using the quota sampling estimator changes the parameter that we are estimating.

Focusing just on the potential bias reduction, the difference in bias between the baseline model and quota sampling can be written as

$$\begin{aligned}
|Bias(\hat{\tau}_{k|r_a=1})| - |Bias(\hat{\tau}_{k|r_a=1,q=1})| &= |Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1}) + \\
& Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1})| - \\
& |Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1,q=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=1}) + \\
& Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1,q=1})|
\end{aligned}$$

We can decompose $Bias(\hat{\tau}_{k|r_a=1})$ into a weighted average of the bias in the sub-sample who we would have

surveyed if we had done quota sampling and the bias in the sub-sample who we would have excluded in the quota sampling design. We will denote the number of Wave 2 individuals who would have been excluded under quota sampling as $e_a = n_a - g_a$. Likewise, we will denote the number of Wave 1 individuals who would have been excluded under quota sampling as $e_b = n_b - g_b$.

We then have

$$\begin{aligned}
Bias(\hat{\tau}_{k|r_a=1}) &= Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1}) \\
Bias(\hat{\tau}_{k|r_a=1}) &= \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikb|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1} + \\
&\quad \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikbc|r_a=1} - \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikb|r_a=1} + \\
&\quad \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikc|r_a=1} - \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikbc|r_a=1} + \\
&\quad \bar{\epsilon}_{kt|r_a=1} - \bar{\epsilon}_{kb|r_b=1}
\end{aligned}$$

which we can separate into

$$\begin{aligned}
Bias(\hat{\tau}_{k|r_a=1}) &= \frac{1}{n_a} \sum_{i=1}^{g_a} y_{ikb|r_a=1,q=1} + \frac{1}{n_a} \sum_{i=1}^{e_a} y_{ikb|r_a=1,q=0} - \left(\frac{1}{n_b} \sum_{i=1}^{g_b} y_{ikb|r_b=1,q=1} + \frac{1}{n_b} \sum_{i=1}^{e_b} y_{ikb|r_b=1,q=0} \right) + \\
&\quad \frac{1}{n_a} \sum_{i=1}^{g_a} y_{ikbc|r_a=1,q=1} + \frac{1}{n_a} \sum_{i=1}^{e_a} y_{ikbc|r_a=1,q=0} - \left(\frac{1}{n_a} \sum_{i=1}^{g_a} y_{ikb|r_a=1,q=1} + \frac{1}{n_a} \sum_{i=1}^{e_a} y_{ikb|r_a=1,q=0} \right) + \\
&\quad \frac{1}{n_a} \sum_{i=1}^{g_a} y_{ikc|r_a=1,q=1} + \frac{1}{n_a} \sum_{i=1}^{e_a} y_{ikc|r_a=1,q=0} - \left(\frac{1}{n_a} \sum_{i=1}^{g_a} y_{ikbc|r_a=1,q=1} + \frac{1}{n_a} \sum_{i=1}^{e_a} y_{ikbc|r_a=1,q=0} \right) + \\
&\quad \frac{1}{n_a} \sum_{i=1}^{g_a} y_{ikto|r_a=1,q=1} + \frac{1}{n_a} \sum_{i=1}^{e_a} y_{ikto|r_a=1,q=0} - \left(\frac{1}{n_a} \sum_{i=1}^{g_a} y_{ikt|r_a=1,q=1} + \frac{1}{n_a} \sum_{i=1}^{e_a} y_{ikt|r_a=1,q=0} \right) - \\
&\quad \frac{1}{n_b} \sum_{i=1}^{g_b} y_{ikbo|r_b=1,q=1} + \frac{1}{n_b} \sum_{i=1}^{e_b} y_{ikbo|r_b=1,q=0} - \left(\frac{1}{n_b} \sum_{i=1}^{g_b} y_{ikb|r_b=1,q=1} + \frac{1}{n_b} \sum_{i=1}^{e_b} y_{ikb|r_b=1,q=0} \right)
\end{aligned}$$

We can simplify this expression as follows

$$\begin{aligned}
Bias(\hat{\tau}_{k|r_a=1}) &= \left(\frac{g_a}{n_a}\right) \left(\bar{y}_{ikb|r_a=1,q=1} - \left(\frac{g_b n_a}{g_a n_b}\right) \bar{y}_{ikb|r_b=1,q=1}\right) + \\
&\quad \left(\frac{e_a}{n_a}\right) \left(\bar{y}_{ikb|r_a=1,q=0} - \left(\frac{e_b n_a}{e_a n_b}\right) \bar{y}_{ikb|r_b=1,q=0}\right) + \\
&\quad \left(\frac{g_a}{n_a}\right) Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a}\right) Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=0}) \\
&\quad \left(\frac{g_a}{n_a}\right) \left(\bar{\epsilon}_{kt|r_a=1,q=1} - \left(\frac{g_b n_a}{g_a n_b}\right) \bar{\epsilon}_{kb|r_b=1,q=1}\right) + \\
&\quad \left(\frac{e_a}{n_a}\right) \left(\bar{\epsilon}_{kt|r_a=1,q=0} - \left(\frac{e_b n_a}{e_a n_b}\right) \bar{\epsilon}_{kb|r_b=1,q=0}\right) \\
Bias(\hat{\tau}_{k|r_a=1}) &= \left(\frac{g_a}{n_a}\right) \left(\bar{y}_{ikb|r_a=1,q=1} - \bar{y}_{ikb|r_b=1,q=1} + \bar{y}_{ikb|r_b=1,q=1} - \left(\frac{g_b n_a}{g_a n_b}\right) \bar{y}_{ikb|r_b=1,q=1}\right) + \\
&\quad \left(\frac{e_a}{n_a}\right) \left(\bar{y}_{ikb|r_a=1,q=0} - \bar{y}_{ikb|r_b=1,q=0} + \bar{y}_{ikb|r_b=1,q=0} - \left(\frac{e_b n_a}{e_a n_b}\right) \bar{y}_{ikb|r_b=1,q=0}\right) + \\
&\quad \left(\frac{g_a}{n_a}\right) Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a}\right) Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a}\right) \left(\bar{\epsilon}_{kt|r_a=1,q=1} - \bar{\epsilon}_{kb|r_b=1,q=1} + \bar{\epsilon}_{kb|r_b=1,q=1} - \left(\frac{g_b n_a}{g_a n_b}\right) \bar{\epsilon}_{kb|r_b=1,q=1}\right) + \\
&\quad \left(\frac{e_a}{n_a}\right) \left(\bar{\epsilon}_{kt|r_a=1,q=0} - \bar{\epsilon}_{kb|r_b=1,q=0} + \bar{\epsilon}_{kb|r_b=1,q=0} - \left(\frac{e_b n_a}{e_a n_b}\right) \bar{\epsilon}_{kb|r_b=1,q=0}\right) \\
Bias(\hat{\tau}_{k|r_a=1}) &= \left(\frac{g_a}{n_a}\right) Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{g_a}{n_a}\right) \left(\bar{y}_{ikb|r_b=1,q=1} - \left(\frac{g_b n_a}{g_a n_b}\right) \bar{y}_{ikb|r_b=1,q=1}\right) + \\
&\quad \left(\frac{e_a}{n_a}\right) Bias_{\mathbf{X}}(\hat{\tau}_{k|r_a=1,q=0}) + \left(\frac{e_a}{n_a}\right) \left(\bar{y}_{ikb|r_b=1,q=0} - \left(\frac{e_b n_a}{e_a n_b}\right) \bar{y}_{ikb|r_b=1,q=0}\right) + \\
&\quad \left(\frac{g_a}{n_a}\right) Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias_{\mathbf{T}}(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a}\right) Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias_{\mathbf{A}}(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a}\right) Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{g_a}{n_a}\right) \left(\bar{\epsilon}_{kb|r_b=1,q=1} - \left(\frac{g_b n_a}{g_a n_b}\right) \bar{\epsilon}_{kb|r_b=1,q=1}\right) + \\
&\quad \left(\frac{e_a}{n_a}\right) Bias_{\mathbf{M}}(\hat{\tau}_{k|r_a=1,q=0}) + \left(\frac{e_a}{n_a}\right) \left(\bar{\epsilon}_{kb|r_b=1,q=0} - \left(\frac{e_b n_a}{e_a n_b}\right) \bar{\epsilon}_{kb|r_b=1,q=0}\right)
\end{aligned}$$

$$\begin{aligned}
Bias(\hat{\tau}_{k|r_a=1}) &= \left(\frac{g_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a}\right) \bar{y}_{ikb|r_b=1,q=1} - \left(\frac{g_b}{n_b}\right) \bar{y}_{ikb|r_b=1,q=1} + \left(\frac{e_a}{n_a}\right) \bar{y}_{ikb|r_b=1,q=0} - \left(\frac{e_b}{n_b}\right) \bar{y}_{ikb|r_b=1,q=0} + \\
&\quad \left(\frac{g_a}{n_a}\right) \bar{c}_{kb|r_b=1,q=1} - \left(\frac{g_b}{n_b}\right) \bar{c}_{kb|r_b=1,q=1} + \left(\frac{e_a}{n_a}\right) \bar{c}_{kb|r_b=1,q=0} - \left(\frac{e_b}{n_b}\right) \bar{c}_{kb|r_b=1,q=0} \\
Bias(\hat{\tau}_{k|r_a=1}) &= \left(\frac{g_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a}\right) \bar{y}_{ikbo|r_b=1,q=1} - \left(\frac{g_b}{n_b}\right) \bar{y}_{ikbo|r_b=1,q=1} + \left(\frac{e_a}{n_a}\right) \bar{y}_{ikbo|r_b=1,q=0} - \left(\frac{e_b}{n_b}\right) \bar{y}_{ikbo|r_b=1,q=0} \\
Bias(\hat{\tau}_{k|r_a=1}) &= \left(\frac{g_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a}\right) \bar{y}_{ikbo|r_b=1,q=1} - \left(\frac{g_b}{n_b}\right) \bar{y}_{ikbo|r_b=1,q=1} + \left(1 - \frac{g_a}{n_a}\right) \bar{y}_{ikbo|r_b=1,q=0} - \left(1 - \frac{g_b}{n_b}\right) \bar{y}_{ikbo|r_b=1,q=0} \\
Bias(\hat{\tau}_{k|r_a=1}) &= \left(\frac{g_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=0}) + \\
&\quad \left(\frac{g_a}{n_a} - \frac{g_b}{n_b}\right) \bar{y}_{ikbo|r_b=1,q=1} - \left(\frac{g_a}{n_a} - \frac{g_b}{n_b}\right) \bar{y}_{ikbo|r_b=1,q=0} \\
Bias(\hat{\tau}_{k|r_a=1}) &= \left(\frac{g_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=0}) + \tag{A2} \\
&\quad \left(\frac{g_a}{n_a} - \frac{g_b}{n_b}\right) (\bar{y}_{ikbo|r_b=1,q=1} - \bar{y}_{ikbo|r_b=1,q=0}) \tag{A3}
\end{aligned}$$

Therefore, we have shown that the bias in the standard estimator in the baseline model is simply the weighted average of the bias in the estimate from a quota sample and the bias for the sub-sample that would be excluded, along with a residual correction factor.

Quota sampling will then decrease bias if and only if

$$|Bias(\hat{\tau}_{k|r_a=1,q=1})| < |Bias(\hat{\tau}_{k|r_a=1})|$$

or (utilizing Lines A2-A3)

$$\begin{aligned}
\left| Bias(\hat{\tau}_{k|r_a=1,q=1}) \right| &< \left| \left(\frac{g_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=1}) + \left(\frac{e_a}{n_a}\right) Bias(\hat{\tau}_{k|r_a=1,q=0}) + \right. \\
&\quad \left. \left(\frac{g_a}{n_a} - \frac{g_b}{n_b}\right) (\bar{y}_{ikbo|r_b=1,q=1} - \bar{y}_{ikbo|r_b=1,q=0}) \right|
\end{aligned}$$

□

Examples of How Quota Sampling Could Amplify Bias

We will begin with a hypothetical phone survey that was carried out in a town before and after an important event. In Wave 1, the survey firm was able to meet its quotas without needing to call anyone twice. However, before the end of Wave 2, the survey firm ran out of people to call and began redialing numbers. Without the quota constraints, the firm may have been able to contact a sufficient number of people without making multiple attempts to reach a single individual. However, the quota constraints in this example would lead to a Wave 2 sample with (on average) harder-to-reach individuals than the Wave 1 sample. These harder-to-reach individuals might differ in many ways from the easier-to-reach individuals, even after conditioning on the covariates that were balanced through the quotas. As such, quota sampling could either reduce or amplify bias compared to convenience sampling, depending on the relationship between the covariates and the potential outcomes.

We can next consider a hypothetical example involving an online survey. In Wave 1, the survey company is able to meet its quotas without an issue. However, in Wave 2, the quota constraints make it difficult for the survey company to obtain a sufficiently large sample. For this reason, the survey company has to work harder, either by advertising the survey more broadly or by offering to pay respondents more. This change in sampling procedures could lead to large demographic differences between the Wave 1 and Wave 2 respondents, for instance on unobservables. Meanwhile, convenience sampling would have resulted in some imbalance on the factors that quota sampling did balance. However, balance on other factors might be much better under convenience sampling than quota sampling. Whether quota sampling or convenience sampling would lead to greater bias would depend on the relationship between the imbalanced factors in each design and the potential outcomes.

4 Rolling Cross-Sections

Under this design, researchers start with a large group of individuals and randomly assign them to be asked to complete the survey in either Wave 1 or Wave 2. Some of the individuals complete the survey and others do not, sometimes because they are never successfully contacted. We can think about our sample as including a group of always-responders who will complete the survey if asked in either Wave 1 or Wave 2, as well as a group of sometimes-responders who would complete the survey in either Wave 1 or Wave 2 but not both. There may also be some never-responders, but we will put them aside for this analysis since they are inaccessible to us. Let n_w denote the total number of always-responders and sometimes-responders. Further, we can denote the number who actually complete the survey in Wave 1 as n_b and the number who actually complete the survey in Wave 2 as n_a . Among the n_b Wave 1 respondents, we will use n_b^* to denote the number of Wave 1 always-responders and m_b^* to

denote the number of Wave 1 sometimes-responders (who would not have completed the survey if we had tried to ask them in Wave 2). Likewise, among the n_a Wave 2 respondents, we will use n_a^* to denote the number of Wave 2 always-responders and m_a^* to denote the number of Wave 2 sometimes-responders (who would not have completed the survey if we had tried to ask them in Wave 1). We will denote the total number of respondents by n and the total number of always-responders by n^* .

For clarity, we have these relationships:

$$n^* \leq n_w$$

$$n_b + n_a = n$$

$$n_b^* + m_b^* = n_b$$

$$n_a^* + m_a^* = n_a$$

$$n_b^* + n_a^* = n^*$$

Also note that n_w and n^* are parameters that do not depend on the randomization. Meanwhile, n_a , n_b , n_a^* , n_b^* , m_a^* , and m_b^* are all random variables that depend on the randomization.

We can denote whether individual i is an always-responder (instead of a sometimes-responder) by $u_i \in \{0, 1\}$. We can also continue to denote whether an individual completed the survey in Wave 1 by $r_{ib} \in \{0, 1\}$ and whether they completed the survey in Wave 2 by $r_{ia} \in \{0, 1\}$. Further, we will let $s_i \in \{0, 1\}$ denote whether individual i is a sometimes-responder, $s_{i1} \in \{0, 1\}$ denote whether individual i is a sometimes-responder who would only complete the survey in Wave 1, and $s_{i2} \in \{0, 1\}$ denote whether individual i is a sometimes-responder who would only complete the survey in Wave 2. We will also let m_b denote the number of sometimes-responders who would complete the survey if they were assigned to take it in Wave 1 and m_a denote the number of sometimes-responders who would complete the survey if they were assigned to take it in Wave 2. Further, let $w_i \in \{0, 1\}$ denote whether individual i would complete the survey in at least one of the two waves, $w_{i1} \in \{0, 1\}$ denote whether individual i would complete the survey if assigned to Wave 1, and $w_{i2} \in \{0, 1\}$ denote whether individual i would complete the survey if assigned to Wave 2. Then the number of individuals who would complete the survey if assigned to Wave 1 can be written as $w_b = n^* + m_b = \sum_i^N w_{i1}$ and the number who would complete the survey if assigned to Wave 2 can be written as $w_a = n^* + m_a = \sum_i^N w_{i2}$. For clarity, note that w_a , w_b , m_a , and m_b are parameters and that $n^* + m_a + m_b = n_w$.

We might be tempted to think that the causal parameter of interest is the average treatment effect for the Wave 2 respondents:

$$\frac{1}{n_a} \sum_{i=1}^{n_a} (y_{ikt|r_a=1} - y_{ikc|r_a=1}).$$

However, this value is a random variable, not a parameter, since n_a is a random variable. Instead, there are two causal parameters that we might want to estimate. The first is the average treatment effect for the always-responders from Waves 1 and 2:

$$\bar{\tau}_{k|u=1} = \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1})$$

The second is the average treatment effect for the combined group of always-responders, sometimes-responders who we sampled, and sometimes-responders who we might have sampled but did not due to the randomization:

$$\bar{\tau}_{k|w=1} = \frac{1}{n_w} \sum_{i=1}^{n_w} (y_{ikt|w=1} - y_{ikc|w=1})$$

The statistic that we will use to estimate both parameters is the average difference in reported answers between the n_a respondents who complete the survey in Wave 2 and the n_b respondents who complete it in Wave 1.

$$\hat{\tau}_{k|u=1} = \hat{\tau}_{k|w=1} = \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}$$

4.1 Proof of Proposition 3

Proposition 3. *When estimating $\bar{\tau}_{k|u=1}$, the bias in $\hat{\tau}_{k|u=1}$ can be written as*

$$Bias(\hat{\tau}_{k|u=1}) = Bias_S(\hat{\tau}_{k|u=1}) + Bias_T(\hat{\tau}_{k|u=1}) + Bias_A(\hat{\tau}_{k|u=1}) + Bias_M(\hat{\tau}_{k|u=1})$$

where

$$Bias_S(\hat{\tau}_{k|u=1}) = \frac{m_a}{w_a} (\bar{y}_{ikt|s_2=1} - \bar{y}_{ikt|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1})$$

$$Bias_T(\hat{\tau}_{k|u=1}) = \bar{y}_{ikc|u=1} - \bar{y}_{ikb|u=1}$$

$$Bias_A(\hat{\tau}_{k|u=1}) = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1}$$

$$Bias_M(\hat{\tau}_{k|u=1}) = \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

Proof. When estimating $\bar{\tau}_{k|u=1}$, the bias in $\hat{\tau}_{k|u=1}$ is

$$\begin{aligned} \text{Bias}(\hat{\tau}_{k|u=1}) &= E[\hat{\tau}_{k|u=1}] - \bar{\tau}_{k|u=1} \\ &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikto|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikbo|r_b=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) \end{aligned}$$

Taking advantage of the fact that $y_{ikto} = y_{ikt} + \epsilon_{ikt}$ and $y_{ikbo} = y_{ikb} + \epsilon_{ikb}$, we get

$$\begin{aligned} \text{Bias}(\hat{\tau}_{k|u=1}) &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikto|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikbo|r_b=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) \\ &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} (y_{ikt|r_a=1} + \epsilon_{ikt|r_a=1}) - \frac{1}{n_b} \sum_{i=1}^{n_b} (y_{ikb|r_b=1} + \epsilon_{ikb|r_b=1})\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) \\ &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1}\right] + \bar{\epsilon}_{ikt|w_2=1} - E\left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right] - \bar{\epsilon}_{ikb|w_1=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) \end{aligned}$$

In the last line, we utilize the fact that the n_a values of $\epsilon_{ikt|r_a=1}$ are a random sample from the $\epsilon_{ikt|w_2=1}$ values, and likewise the n_b values of $\epsilon_{ikb|r_b=1}$ are a random sample from the $\epsilon_{ikb|w_1=1}$ values.

We can further rewrite the overall bias term as

$$\begin{aligned} \text{Bias}(\hat{\tau}_{k|u=1}) &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1}\right] - E\left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) + \\ &\quad \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\ &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1}\right] - E\left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} + \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} + \\ &\quad \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\ &= \left(E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1}\right) + \left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - E\left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right]\right) + \quad (\text{A4}) \end{aligned}$$

$$\bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (\text{A5})$$

We can begin by focusing on the first of the two differences in Line A4. We can break this expression down as follows:

$$\begin{aligned}
E \left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1} \right] - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} &= E \left[\frac{1}{n_a} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} + \frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} \\
&= E \left[\frac{1}{n_a} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] + E \left[\frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} \\
&= E \left[\frac{1}{n_a} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} + E \left[\frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right]
\end{aligned} \tag{A6}$$

Focusing on the first term in Line A6, note that

$$\begin{aligned}
E \left[\frac{1}{n_a} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] &= E \left[\frac{n_a^*}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \\
&= E \left[\frac{n_a^* + (n_a - n_a)}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \\
&= E \left[\frac{n_a^* - n_a}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] + E \left[\frac{n_a}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \\
&= E \left[\frac{-m_a^*}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] + E \left[\frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right]
\end{aligned} \tag{A7}$$

The second term in Line A7 is the expected value of a random sample of n_a^* draws from the $y_{ikt|u=1}$ values (the y_{ikt} of the always-responders). Therefore, it equals the mean of the $y_{ikt|u=1}$ values.

$$E \left[\frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} \tag{A8}$$

Combining Lines A4-A5, A6, A7, and A8, we get

$$\begin{aligned}
Bias(\hat{\tau}_{k|u=1}) &= \left(E \left[\frac{-m_a^*}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] + \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} + E \left[\frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] \right) + \\
&\quad \left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - E \left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1} \right] \right) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= \left(E \left[\frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - E \left[\frac{m_a^*}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \right) +
\end{aligned} \tag{A9}$$

$$\left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - E \left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1} \right] \right) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \tag{A10}$$

We can now focus on the first difference in Line A10. We can rewrite this expression as

$$\begin{aligned}
\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - E \left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1} \right] &= \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \left(E \left[\frac{1}{n_b} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] + E \left[\frac{1}{n_b} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] \right) \\
&= \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - E \left[\frac{1}{n_b} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] - E \left[\frac{1}{n_b} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right]
\end{aligned} \tag{A11}$$

The last term in Line A11 can be written as

$$\begin{aligned}
E \left[\frac{1}{n_b} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] &= E \left[\frac{n_b^*}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] \\
&= E \left[\frac{n_b^* + (n_b - n_b^*)}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] \\
&= E \left[\frac{n_b^* - n_b}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] + E \left[\frac{n_b}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] \\
&= E \left[\frac{-m_b^*}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] + E \left[\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right]
\end{aligned} \tag{A12}$$

Similar to what we did in Line A8, we can note that the second term in Line A12 is the expected value of a sample of n_b^* draws from the $y_{ikb|u=1}$ values (the y_{ikb} of the always-responders). Therefore, it equals the mean of the $y_{ikb|u=1}$ values.

$$E \left[\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} \tag{A13}$$

Combining Lines A9-A10, A11, A12, and A13, we get

$$\begin{aligned}
Bias(\hat{\tau}_{k|u=1}) &= \left(E \left[\frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - E \left[\frac{m_a^*}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \right) + \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \\
& E \left[\frac{1}{n_b} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] - \left(E \left[\frac{-m_b^*}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] + \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} \right) + \\
& \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= \left(E \left[\frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - E \left[\frac{m_a^*}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \right) + \tag{A14}
\end{aligned}$$

$$\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} + \tag{A15}$$

$$\left(E \left[\frac{m_b^*}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] - E \left[\frac{1}{n_b} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] \right) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \tag{A16}$$

The expression in Line A15 is the average difference between the Wave 2 truthful answers of the always-responders in the counterfactual world where the event did not happen and their Wave 1 truthful answers in the world where the event did happen. As we did in Section 2, we can decompose this term into the bias caused by temporal factors between Waves 1 and 2 and the bias caused by anticipatory factors.

$$\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} = \bar{y}_{ikc|u=1} - \bar{y}_{ikb|u=1} \tag{A17}$$

$$= Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) \tag{A18}$$

where

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikc|u=1} - \bar{y}_{ikbc|u=1} \tag{A19}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1} \tag{A20}$$

Therefore, we can write the overall bias term as

$$Bias(\hat{\tau}_{k|u=1}) = \left(E \left[\frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - E \left[\frac{m_a^*}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \right) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + \quad (\text{A21})$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \left(E \left[\frac{m_b^*}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] - E \left[\frac{1}{n_b} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] \right) + \quad (\text{A22})$$

$$\bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (\text{A23})$$

We will start with the first difference in Line A21. Note that

$$\begin{aligned} E \left[\frac{1}{n_a} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - E \left[\frac{m_a^*}{n_a n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] &= E \left[\frac{m_a^*}{n_a} \left(\frac{1}{m_a^*} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right) - \frac{m_a^*}{n_a} \left(\frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right) \right] \\ &= E \left[\frac{m_a^*}{n_a} \left(\frac{1}{m_a^*} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} - \frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right) \right] \end{aligned} \quad (\text{A24})$$

So the expression inside the parentheses in Line A24 is just the difference of two averages. The outside weight $\frac{m_a^*}{n_a}$ is the proportion of Wave 2 respondents who are sometimes-responders. Inside the parentheses, the first average is the mean y_{ikt} value of the Wave 2 respondents who are sometimes-responders, and the second average is the mean y_{ikt} value of the Wave 2 respondents who are always-responders.

Therefore, the first average inside the parentheses in Line A24 is the average of a random draw of the m_a^* sometimes responders who would only complete the survey in Wave 2. We can use p to denote the probability that an individual will initially be randomized to be contacted in Wave 1, making $1 - p$ their initial probability of being contacted in Wave 2. Then the expected numbers of always-responders and sometimes-responders who will complete the survey in Wave 1 and Wave 2 are

$$E[n_b^*] = pn^*$$

$$E[n_a^*] = (1 - p)n^*$$

$$E[m_b^*] = pm_b$$

$$E[m_a^*] = (1 - p)m_a$$

Similarly, the expected numbers of Wave 1 and Wave 2 respondents are

$$E [n_b] = E [n_b^* + m_b^*] = E [n_b^*] + E [m_b^*] = p (n^* + m_b)$$

$$E [n_a] = E [n_a^* + m_a^*] = E [n_a^*] + E [m_a^*] = (1 - p) (n^* + m_a)$$

Also, the expected proportions of sometimes-responders in Waves 1 and 2 are

$$E \left[\frac{m_b^*}{n_b} \right] = \frac{m_b}{w_b}$$

$$E \left[\frac{m_a^*}{n_a} \right] = \frac{m_a}{w_a}$$

Returning to the expression in Line A24, the proportion of Wave 2 respondents who are sometimes-responders is statistically independent of the mean y_{ikt} value of these sometimes-responders. Likewise, it is statistically independent of the mean y_{ikt} value of the Wave 2 respondents who are always-responders. We therefore have

$$\begin{aligned} E \left[\frac{m_a^*}{n_a} \left(\frac{1}{m_a^*} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} - \frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right) \right] &= E \left[\frac{m_a^*}{n_a} \right] E \left[\frac{1}{m_a^*} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} - \frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \\ &= \frac{m_a}{w_a} \left(E \left[\frac{1}{m_a^*} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - E \left[\frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \right) \end{aligned} \quad (\text{A25})$$

Inside the parentheses of Line A25, the first term is the average of m_a^* random draws from the y_{ikt} values of the m_a sometimes-responders who would only complete the survey in Wave 2. Similarly, the second term is the average of n_a^* random draws from the y_{ikt} values of the n^* always-responders. Therefore, we have

$$E \left[\frac{1}{m_a^*} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] = \frac{1}{m_a} \sum_{i=1}^{m_a} y_{ikt|s_2=1}$$

and

$$E \left[\frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1}$$

We can then write

$$\frac{m_a}{w_a} \left(E \left[\frac{1}{m_a^*} \sum_{i=1}^{m_a^*} y_{ikt|r_a=1, s_2=1} \right] - E \left[\frac{1}{n_a^*} \sum_{i=1}^{n_a^*} y_{ikt|r_a=1, u=1} \right] \right) = \frac{m_a}{w_a} \left(\frac{1}{m_a} \sum_{i=1}^{m_a} y_{ikt|s_2=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} \right)$$

Substituting this expression into the overall bias term, we get

$$Bias(\hat{\tau}_{k|u=1}) = \frac{m_a}{w_a} \left(\frac{1}{m_a} \sum_{i=1}^{m_a} y_{ikt|s_2=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} \right) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + \quad (A26)$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \left(E \left[\frac{m_b^*}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] - E \left[\frac{1}{n_b} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] \right) + \quad (A27)$$

$$\bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (A28)$$

We can now turn to the difference in Line A27. Similar to before, we can begin by noting that

$$\begin{aligned} E \left[\frac{m_b^*}{n_b n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] - E \left[\frac{1}{n_b} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] &= E \left[\frac{m_b^*}{n_b} \left(\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right) - \frac{m_b^*}{n_b} \left(\frac{1}{m_b^*} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right) \right] \\ &= E \left[\frac{m_b^*}{n_b} \left(\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} - \frac{1}{m_b^*} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right) \right] \end{aligned} \quad (A29)$$

As in Line A24, the expression inside the parentheses in Line A29 is just the difference of two averages. The outside weight $\frac{m_b^*}{n_b}$ is the proportion of Wave 1 respondents who are sometimes-responders. Inside the parentheses, the first average is the mean y_{ikb} value of the Wave 1 respondents who are always-responders, and the second average is the mean y_{ikb} value of the Wave 1 respondents who are sometimes-responders. Since in this context the weight is statistically independent of the averages, as explained earlier, we can write

$$\begin{aligned} E \left[\frac{m_b^*}{n_b} \left(\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} - \frac{1}{m_b^*} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right) \right] &= E \left[\frac{m_b^*}{n_b} \right] E \left[\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} - \frac{1}{m_b^*} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] \\ &= \frac{m_b}{w_b} \left(E \left[\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] - E \left[\frac{1}{m_b^*} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] \right) \end{aligned} \quad (A30)$$

Inside the parentheses of Line A30, the first term is the average of n_b^* random draws from the y_{ikb} values of the n^* always-responders. Likewise, the second term is the average of m_b^* random draws from the y_{ikb} values of the m_b sometimes-responders who would only complete the survey in Wave 1. Therefore, we have

$$E \left[\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1}$$

and

$$E \left[\frac{1}{m_b^*} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] = \frac{1}{m_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1}$$

We can then write

$$\frac{m_b}{w_b} \left(E \left[\frac{1}{n_b^*} \sum_{i=1}^{n_b^*} y_{ikb|r_b=1, u=1} \right] - E \left[\frac{1}{m_b^*} \sum_{i=1}^{m_b^*} y_{ikb|r_b=1, s_1=1} \right] \right) = \frac{m_b}{w_b} \left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} - \frac{1}{m_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1} \right)$$

Substituting this expression into the overall bias term allows us to write

$$\begin{aligned} Bias(\hat{\tau}_{k|u=1}) &= \frac{m_a}{w_a} \left(\frac{1}{m_a} \sum_{i=1}^{m_a} y_{ikt|s_2=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} \right) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \\ &\frac{m_b}{w_b} \left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} - \frac{1}{m_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1} \right) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\ &= \frac{m_a}{w_a} (\bar{y}_{ikt|s_2=1} - \bar{y}_{ikt|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \\ &\frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1}) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\ &= \frac{m_a}{w_a} (\bar{y}_{ikt|s_2=1} - \bar{y}_{ikt|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1}) + \end{aligned} \quad (\text{A31})$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (\text{A32})$$

The first expression in Line A31 is the proportion of possible Wave 2 respondents who are sometimes-responders multiplied by the average difference between these sometimes-responders' y_{ikt} values and the always-responders' y_{ikt} values. The second expression in Line A31 is the proportion of possible Wave 1 respondents who are sometimes-responders multiplied by the average difference between the always-responders' y_{ikb} values and these potential Wave 1 sometimes-responders' y_{ikb} values. Therefore, we can think of the sum of these two expressions as the bias caused by having sometimes-responders in the Wave 1 and Wave 2 samples.

$$Bias_{\mathbf{S}}(\hat{\tau}_{k|u=1}) = \frac{m_a}{w_a} (\bar{y}_{ikt|s_2=1} - \bar{y}_{ikt|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1})$$

We now have

$$Bias(\hat{\tau}_{k|u=1}) = Bias_{\mathbf{S}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (\text{A33})$$

The expression $\bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$ is the potential bias caused by differential misreporting in the answers of the

possible Wave 1 and Wave 2 respondents. We can define it as

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) = \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

In sum, we can write the overall bias as

$$Bias(\hat{\tau}_{k|u=1}) = Bias_{\mathbf{S}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) \quad (\text{A34})$$

where

$$Bias_{\mathbf{S}}(\hat{\tau}_{k|u=1}) = \frac{m_a}{w_a} (\bar{y}_{ikt|s_2=1} - \bar{y}_{ikt|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1})$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikc|u=1} - \bar{y}_{ikbc|u=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1}$$

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) = \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

□

4.2 Proof of Proposition 4

Proposition 4. *When estimating $\bar{\tau}_{k|w=1}$, the bias in $\hat{\tau}_{k|w=1}$ is*

$$Bias(\hat{\tau}_{k|w=1}) = Bias_{\mathbf{S}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|w=1})$$

where,

$$Bias_{\mathbf{S}}(\hat{\tau}_{k|w=1}) = \frac{m_b}{n_w} (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|s_1=1}) + \frac{m_a}{n_w} (\bar{y}_{ikb|s_2=1} - \bar{y}_{ikb|w_1=1})$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) = \bar{y}_{ikc|w=1} - \bar{y}_{ikbc|w=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) = \bar{y}_{ikbc|w=1} - \bar{y}_{ikb|w=1}$$

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|w=1}) = \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

Proof. Begin by recalling that the average treatment effect of the combined group of always-responders, sometimes-responders who we sampled, and sometimes-responders who we might have sampled but did not due to the randomization is written as

$$\bar{\tau}_{k|w=1} = \frac{1}{n_w} \sum_{i=1}^{n_w} (y_{ikt|w=1} - y_{ikc|w=1}) \quad (\text{A35})$$

To estimate this parameter, we will use the same estimator as before: the average difference in reported outcomes between the n_a respondents who complete the survey in Wave 2 and the n_b respondents who complete it in Wave 1.

$$\hat{\tau}_{k|w=1} = \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikto|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikbo|r_b=1}$$

The bias in this estimator is therefore

$$\begin{aligned} \text{Bias}(\hat{\tau}_{k|w=1}) &= E[\hat{\tau}_{k|w=1}] - \bar{\tau}_{k|w=1} \\ &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikto|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikbo|r_b=1}\right] - \frac{1}{n_w} \sum_{i=1}^{n_w} (y_{ikt|w=1} - y_{ikc|w=1}) \\ &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} (y_{ikt|r_a=1} + \epsilon_{ikt|r_a=1}) - \frac{1}{n_b} \sum_{i=1}^{n_b} (y_{ikb|r_b=1} + \epsilon_{ikb|r_b=1})\right] - \frac{1}{n_w} \sum_{i=1}^{n_w} (y_{ikt} - y_{ikc}) \\ &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1}\right] + \bar{\epsilon}_{ikt|w_2=1} - E\left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right] - \bar{\epsilon}_{ikb|w_1=1} - \frac{1}{n_w} \sum_{i=1}^{n_w} (y_{ikt|w=1} - y_{ikc|w=1}) \\ &= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1}\right] - E\left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right] - \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikt|w=1} + \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikc|w=1} + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\ &= \left(E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1}\right] - \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikt|w=1}\right) + \left(\frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikc|w=1} - E\left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right]\right) + \end{aligned} \quad (\text{A36})$$

$$\bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (\text{A37})$$

Note that, like before, $w_{i1} \in \{0, 1\}$ denotes whether an individual would complete the survey if assigned to Wave 1 and $w_{i2} \in \{0, 1\}$ denotes whether they would complete the survey if assigned to Wave 2. Similarly, $w_i \in \{0, 1\}$ denotes whether individual i would complete the survey in at least one of the two waves. Likewise, $w_b = n^* + m_b$ denotes the number of individuals who would complete the survey if asked to do it in Wave 1, and $w_a = n^* + m_a$ denotes the number of individuals who would complete the survey if asked to do it in Wave 2. Focusing on the first difference in Line A36, we can rewrite the expression as

$$\begin{aligned}
E \left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikt|r_a=1} \right] - \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikt|w=1} &= \frac{1}{w_a} \sum_{i=1}^{w_a} y_{ikt|w_2=1} - \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikt|w=1} \\
&= \bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|w=1}
\end{aligned} \tag{A38}$$

Similarly, the second difference in Line A36 can be rewritten as

$$\begin{aligned}
\frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikc|w=1} - E \left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1} \right] &= \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikc|w=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} \\
&= \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikc|w=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} - \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \\
&= \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikc|w=1} - \left(\frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \right) +
\end{aligned} \tag{A39}$$

$$\frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \tag{A40}$$

We can rewrite the expression inside the parentheses in Line A39 as

$$\begin{aligned}
\frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} &= \frac{n_w}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \\
&= \frac{n_w + (w_b - w_b)}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \\
&= \frac{n_w - w_b}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{w_b}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \\
&= \frac{n_w - w_b}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \\
&= \frac{n_w - w_b}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikb|w=1}
\end{aligned}$$

Combining Lines A38, A39-A40, and A41, we get

$$\begin{aligned}
Bias(\hat{\tau}_{k|w=1}) &= \bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|w=1} + \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikc|w=1} - \left(\frac{n_w - w_b}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikb|w=1} \right) + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} + \\
&\quad \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|w=1}) + \left(\frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikc|w=1} - \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikb|w=1} \right) + \frac{w_b - n_w}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} + \\
&\quad \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|w=1}) + (\bar{y}_{ikc|w=1} - \bar{y}_{ikb|w=1}) + \frac{w_b - n_w}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} + \\
&\quad \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}
\end{aligned}$$

Similar to our discussion in Section 2 of the paper, this difference $\bar{y}_{ikc|w=1} - \bar{y}_{ikb|w=1}$ can be decomposed into the bias from temporal factors and the bias from anticipatory factors. Thus, we can write

$$\bar{y}_{ikc|w=1} - \bar{y}_{ikb|w=1} = Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1})$$

where

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) = \bar{y}_{ikc|w=1} - \bar{y}_{ikbc|w=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) = \bar{y}_{ikbc|w=1} - \bar{y}_{ikb|w=1}$$

We can now write the overall bias term as:

$$Bias(\hat{\tau}_{k|w=1}) = (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|w=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) + \tag{A41}$$

$$\left(\frac{w_b - n_w}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \right) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \tag{A42}$$

The first difference in Line A42 can be written as

$$\begin{aligned}
\frac{w_b - n_w}{w_b n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} &= \frac{1}{n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} \\
&= \frac{1}{n_w} \sum_{i=1}^{w_b} y_{ikb|w_1=1} + \frac{1}{n_w} \sum_{i=1}^{m_a} y_{ikb|s_2=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} \\
&= \frac{1}{n_w} \sum_{i=1}^{n_w} y_{ikb|w=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} \\
&= \bar{y}_{ikb|w=1} - \bar{y}_{ikb|w_1=1}
\end{aligned}$$

We can now write the overall bias term as

$$\begin{aligned}
Bias(\hat{\tau}_{k|w=1}) &= (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|w=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) + (\bar{y}_{ikb|w=1} - \bar{y}_{ikb|w_1=1}) + \\
&\quad \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|w=1}) + (\bar{y}_{ikb|w=1} - \bar{y}_{ikb|w_1=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) + \quad (A43)
\end{aligned}$$

$$\bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (A44)$$

The differences within the two sets of parentheses in Line A43 come from not being able to see any y_{ikto} values for the Wave 1 sometimes-responders nor any of the y_{ikbo} values for the Wave 2 sometimes-responders. We can think of this bias as arising from having sometimes-responders in our sample who differ in systematic ways from the always-responders and the sometimes-responders who answer the survey in the other wave. We can rewrite this bias term as

$$(\bar{y}_{ikt|w_2=1} - \bar{y}_{ikb|w_1=1}) - (\bar{y}_{ikt|w=1} - \bar{y}_{ikb|w=1}) = (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikb|w_1=1}) - \left(\frac{w_a}{n_w}\right) \bar{y}_{ikt|w_2=1} - \left(\frac{m_b}{n_w}\right) \bar{y}_{ikt|s_1=1} + \quad (A45)$$

$$\left(\frac{w_b}{n_w}\right) \bar{y}_{ikb|w_1=1} + \left(\frac{m_a}{n_w}\right) \bar{y}_{ikb|s_2=1} \quad (A46)$$

$$= \frac{m_b}{n_w} (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|s_1=1}) + \frac{m_a}{n_w} (\bar{y}_{ikb|s_2=1} - \bar{y}_{ikb|w_1=1}) \quad (A47)$$

We can label this bias

$$Bias_{\mathbf{S}}(\hat{\tau}_{k|w=1}) = \frac{m_b}{n_w} (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|s_1=1}) + \frac{m_a}{n_w} (\bar{y}_{ikb|s_2=1} - \bar{y}_{ikb|w_1=1})$$

We can now write the overall bias term as

$$Bias(\hat{\tau}_{k|w=1}) = Bias_{\mathbf{S}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) + \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

As in the previous proof, $\bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$ is the potential bias caused by differential misreporting in the answers of the possible Wave 1 and Wave 2 respondents. We can define it as

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|w=1}) = \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

In sum, we can write the overall bias as

$$Bias(\hat{\tau}_{k|w=1}) = Bias_{\mathbf{S}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|w=1}) \quad (\text{A48})$$

where

$$Bias_{\mathbf{S}}(\hat{\tau}_{k|w=1}) = \frac{m_b}{n_w} (\bar{y}_{ikt|w_2=1} - \bar{y}_{ikt|s_1=1}) + \frac{m_a}{n_w} (\bar{y}_{ikb|s_2=1} - \bar{y}_{ikb|w_1=1})$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) = \bar{y}_{ikc|w=1} - \bar{y}_{ikbc|w=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}) = \bar{y}_{ikbc|w=1} - \bar{y}_{ikb|w=1}$$

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|w=1}) = \bar{\epsilon}_{ikt|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

□

Before proceeding, we can consider the special case where there are no always-responders. We can think of this scenario as the baseline model but when the parameter that we are estimating is the average treatment effect for all Wave 1 and Wave 2 respondents. The $Bias_{\mathbf{S}}(\hat{\tau}_{k|w=1})$ term becomes a weighted average of the pre-event and post-event differences between the Wave 1 and Wave 2 respondents' truthful answers. Clearly, bias when estimating the average treatment effect for Wave 2 respondents in the baseline model is more straightforward to comprehend than bias when estimating the average treatment effect for both Wave 1 and Wave 2 respondents.

We can now examine how the bias in the estimators $\hat{\tau}_{k|u=1}$ and $\hat{\tau}_{k|w=1}$ from the rolling cross-section design compare to the bias in the baseline model from the paper. The rolling cross-section design trades the bias in demographic differences between Wave 1 and Wave 2 respondents for the bias caused by sometimes-responders. Focusing on Equation A34, we can better understand $Bias_{\mathbf{S}}(\hat{\tau}_{k|u=1})$ by considering the special case where the initial numbers of sometimes-responders in Waves 1 and 2 are the same ($m_a = m_b$). In that case, $\frac{m_a}{w_a} = \frac{m_b}{w_b}$, which we will denote as

$\alpha \leq 1$. This symmetry allows us to rewrite $Bias_S(\hat{\tau}_{k|u=1})$ as

$$\begin{aligned}
Bias_S(\hat{\tau}_{k|u=1}|m_a = m_b) &= \frac{m_a}{w_a} (\bar{y}_{ikt|s_2=1} - \bar{y}_{ikt|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1}) \\
&= \alpha (\bar{y}_{ikt|s_2=1} - \bar{y}_{ikt|u=1}) + \alpha (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1}) \\
&= \alpha (\bar{y}_{ikt|s_2=1} - \bar{y}_{ikb|s_1=1} - \bar{y}_{ikt|u=1} + \bar{y}_{ikb|u=1}) \\
&= \alpha [(\bar{y}_{ikt|s_2=1} - \bar{y}_{ikb|s_1=1}) - (\bar{y}_{ikt|u=1} - \bar{y}_{ikb|u=1})] \tag{A49}
\end{aligned}$$

Note that the terms inside both sets of parentheses in Line A49 resemble our estimator in the baseline model of Section 2 of the paper. In fact, they are equivalent to that estimator in the special case where there is no measurement error. The first term is simply the standard estimator $\hat{\tau}_{k|r_a=1}$ without measurement error on a sample consisting entirely of sometimes-responders. The second term is the same estimator on a sample consisting entirely of always-responders, except in a world where the Wave 1 and Wave 2 individuals are identical on demographic characteristics. Since there is no randomness in the baseline model, we can think of both estimators as the average treatment effect for that sub-sample combined with the corresponding bias term, following from the equation $Bias(\hat{\tau}) = E[\hat{\tau}] - \bar{\tau}$. We can therefore write

$$\begin{aligned}
Bias_S(\hat{\tau}_{k|u=1}|m_a = m_b) &= \alpha [(\bar{\tau}_{k|s=1} + Bias_{\mathbf{X}}(\hat{\tau}_{k|s=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|s=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|s=1})) - \\
&\quad (\bar{\tau}_{k|u=1} + Bias_{\mathbf{X}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}))] \\
&= \alpha [(\bar{\tau}_{k|s=1} + Bias_{\mathbf{X}}(\hat{\tau}_{k|s=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|s=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|s=1})) - \\
&\quad (\bar{\tau}_{k|u=1} + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}))]
\end{aligned}$$

Substituting this expression into Equation A34, we obtain

$$Bias(\hat{\tau}_{k|u=1}|m_a = m_b) = \alpha Bias_{\mathbf{X}}(\hat{\tau}_{k|s=1}) + (\alpha Bias_{\mathbf{T}}(\hat{\tau}_{k|s=1}) + (1 - \alpha) Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1})) + \tag{A50}$$

$$(\alpha Bias_{\mathbf{A}}(\hat{\tau}_{k|s=1}) + (1 - \alpha) Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1})) + Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) + \tag{A51}$$

$$\alpha (\bar{\tau}_{k|s=1} - \bar{\tau}_{k|u=1}) \tag{A52}$$

The expression involving bias from temporal factors is just a weighted average of the temporal bias for the sometimes-responders and always-responders. The same logic holds for the expression involving the bias from anticipatory factors. We also add a new bias term involving the difference in average treatment effect between the sometimes-responders and always-responders.

In sum, the rolling cross section estimator reduces demographic bias, but it also complicates the rest of the overall bias term in ways that could either decrease or enlarge the total bias in this design.

If we instead use Equation A48 and consider the special case where the initial numbers of possible Wave 1 and Wave 2 sometimes-responders are the same ($m_a = m_b$), then the weights we obtain $\frac{m_a}{n_w}$ and $\frac{m_b}{n_w}$ will be equal. In this situation, we can define $\lambda = \frac{m_a}{n_w} = \frac{m_b}{n_w}$. We can then rewrite the equation for $Bias_S(\hat{\tau}_{k|w=1}|m_a = m_b)$ as

$$Bias_S(\hat{\tau}_{k|w=1}|m_a = m_b) = \lambda [(\bar{y}_{ikt|w_2=1} - \bar{y}_{ikb|w_1=1}) - (\bar{y}_{ikt|w=1} - \bar{y}_{ikb|w=1})]$$

Here we are utilizing the way that we wrote $Bias_S(\hat{\tau}_{k|w=1})$ at the beginning of Line A45.

This expression is very similar to what we saw in Line A49. Like before, we can think about each of the two differences inside the brackets as mathematically similar to the estimator from the baseline model, specifically in the case where there is no measurement error. Also like before, we can think of these two estimators as the average treatment effect for that sample combined with the corresponding bias term. We can therefore write

$$Bias_S(\hat{\tau}_{k|w=1}|m_a = m_b) = \lambda [(\bar{\tau}_{k|w_2=1} + Bias_{\mathbf{X}}(\hat{\tau}_{k|w_2=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|w_2=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w_2=1})) - \quad (A53)$$

$$(\bar{\tau}_{k|w=1} + Bias_{\mathbf{X}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}))] \quad (A54)$$

Note that in Line A53, $Bias_{\mathbf{X}}(\hat{\tau}_{k|w_2=1}) = \bar{y}_{ikb|w_2=1} - \bar{y}_{ikb|w_1=1}$. Since the pre-event and post-event samples in Line A54 consist of n_w individuals with exactly the same demographic characteristics, we can drop the $Bias_{\mathbf{X}}(\hat{\tau}_{k|w=1})$ term.

$$Bias_S(\hat{\tau}_{k|w=1}|m_a = m_b) = \lambda [(\bar{\tau}_{k|w_2=1} + Bias_{\mathbf{T}}(\hat{\tau}_{k|w_2=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w_2=1})) - (\bar{\tau}_{k|w=1} + Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1}))]$$

Substituting this expression into Line A48, we get

$$Bias(\hat{\tau}_{k|w=1}|m_a = m_b) = \lambda Bias_{\mathbf{X}}(\hat{\tau}_{k|w_2=1}) + (\lambda Bias_{\mathbf{T}}(\hat{\tau}_{k|w_2=1}) + (1 - \lambda) Bias_{\mathbf{T}}(\hat{\tau}_{k|w=1})) + (\lambda Bias_{\mathbf{A}}(\hat{\tau}_{k|w_2=1}) + (1 - \lambda) Bias_{\mathbf{A}}(\hat{\tau}_{k|w=1})) + Bias_{\mathbf{M}}(\hat{\tau}_{k|w=1}) + \lambda (\bar{\tau}_{k|w_2=1} - \bar{\tau}_{k|w=1})$$

5 Panel Designs

In a panel design, we begin with a group of individuals who have the opportunity to take the same survey in Wave 1 and Wave 2. We can denote whether individual i takes the survey in both waves by $u_i \in \{0, 1\}$ and the total number of respondents who take the survey in both waves as n^* . The causal parameter that we estimate is the average treatment effect of the event on these n^* respondents' truthful answers to question k of the survey:

$$\bar{\tau}_{k|u=1} = \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1})$$

In the above line, we consider $y_{ikt|u=1}$ to be individual i 's truthful answer in the world where they did not complete the survey in Wave 1. We can distinguish this value from $y_{ika|u=1}$, which we use to denote individual i 's truthful answer in the world where they did complete the survey in Wave 1.

The statistic we use to estimate $\bar{\tau}_{k|u=1}$ is

$$\hat{\tau}_{k|u=1} = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikao|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikbo|u=1}$$

In the above line, we use $y_{ikao|u=1}$ to denote individual i 's reported answer in Wave 2 after having already completed the survey in Wave 1.

5.1 Proof of Proposition 5

Proposition 5. *Bias in the panel design can be written as*

$$Bias(\hat{\tau}_{k|u=1}) = Bias_{\mathbf{C}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1})$$

where

$$Bias_{\mathbf{C}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ika|u=1} - \bar{y}_{ikt|u=1}$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikc|u=1} - \bar{y}_{ikbc|u=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1}$$

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) = \bar{\epsilon}_{ka|u=1} - \bar{\epsilon}_{kb|u=1}$$

Proof. The bias in $\hat{\tau}_{k|u=1}$ is just

$$\begin{aligned}
Bias(\hat{\tau}_{k|u=1}) &= E[\hat{\tau}_{k|u=1}] - \bar{\tau}_{k|u=1} \\
&= E\left[\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikao|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikbo|u=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) \\
&= \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikao|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikbo|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} + \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} \\
&= \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ika|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} + \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} + \bar{e}_{ka|u=1} - \bar{e}_{kb|u=1} \\
&= \left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ika|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1}\right) + \left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1}\right) + \bar{e}_{ka|u=1} - \bar{e}_{kb|u=1}
\end{aligned} \tag{A55}$$

The first difference in Line A55 can be thought of as the average difference between the n^* always-responders' Wave 2 truthful answers in the world where they completed the survey in Wave 1 and the world where they did not. In other words, it is the average causal effect of completing the survey in Wave 1 on always-responders' true answers in Wave 2, commonly known as conditioning effects. We denote this bias as

$$Bias_{\mathbf{C}}(\hat{\tau}_{k|u=1}) = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ika|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} = \bar{y}_{ika|u=1} - \bar{y}_{ikt|u=1} \tag{A56}$$

Returning to Line A55, the second difference is similar to what we saw in Equation 5 from Section 2 of the paper. Following what we did in Section 2, we can decompose this expression into bias from temporal factors and bias from anticipatory factors:

$$\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} = Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) \tag{A57}$$

where

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikbc|u=1} = \bar{y}_{ikc|u=1} - \bar{y}_{ikbc|u=1} \tag{A58}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikbc|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1} \tag{A59}$$

Finally, the last difference in Line A55 is just the potential difference in misreporting between Waves 1 and 2.

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) = \bar{e}_{ka|u=1} - \bar{e}_{kb|u=1} \tag{A60}$$

In sum, we can write the bias in the panel design as

$$Bias(\hat{\tau}_{k|u=1}) = Bias_{\mathbf{C}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) \quad (\text{A61})$$

where

$$Bias_{\mathbf{C}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ika|u=1} - \bar{y}_{ikt|u=1} \quad (\text{A62})$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikc|u=1} - \bar{y}_{ikbc|u=1} \quad (\text{A63})$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1} \quad (\text{A64})$$

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) = \bar{\epsilon}_{ka|u=1} - \bar{\epsilon}_{kb|u=1} \quad (\text{A65})$$

□

6 Dual Randomized Survey (DRS) Design

In the DRS design, we have n individuals who complete both surveys. We will let n_a denote the number of individuals who complete Survey A in Wave 2 and n_b denote the number of individuals who complete Survey A in Wave 1, such that $n_a + n_b = n$. Because of the possibility of differential attrition, we can think of our sample as consisting of “always-responders” who would complete both surveys no matter which survey they were assigned to do first and “sometimes-responders” whose participation in Wave 2 depends on which survey they receive in Wave 1. We will denote the number of always-responders by n^* , the number of sometimes-responders who would only complete both surveys if assigned to do Survey A in Wave 2 by m_a , and the number of sometimes-responders who would only complete both surveys if assigned to do Survey A in Wave 1 by m_b . We will also let w_a denote the total possible number of individuals who would complete both surveys if assigned to do Survey A in Wave 2 and w_b denote the total possible number of individuals who would complete both surveys if assigned to do Survey A in Wave 1, such that $w_a = n^* + m_a$ and $w_b = n^* + m_b$. We will use $r_{ia} \in \{0, 1\}$ to denote whether individual i completed Survey A after the event and $r_{ib} \in \{0, 1\}$ to denote whether individual i completed Survey A before the event. Like before, we will use $u_i \in \{0, 1\}$ to denote whether individual i is an always-responder. We will also use $w_{i1} \in \{0, 1\}$ to denote whether individual i would complete both surveys if assigned to do Survey A in Wave 1 and $w_{i2} \in \{0, 1\}$ to denote whether individual i would complete both surveys if assigned to do Survey A in Wave 2.

We will think of each of the individuals in our sample as having a Wave 1 truthful answer (y_{ikb}), a Wave 1 reported answer (y_{ikbo}), a Wave 2 truthful answer in the world where they did not complete Survey B in Wave 1 (y_{ikt}), a Wave 1 truthful answer in the hypothetical world where the event did not happen (y_{ikc}), a Wave 2 truthful answer after having completed Survey B in Wave 1 (y_{ika}), and a Wave 2 reported answer after having completed Survey B in Wave 1 (y_{ikao}).

6.1 Proof of Proposition 6

Proposition 6. *Bias in the DRS design can be written as*

$$Bias(\hat{\tau}_{k|u=1}) = Bias_{\mathbf{D}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1})$$

where

$$Bias_{\mathbf{D}}(\hat{\tau}_{k|u=1}) = \frac{m_a}{w_a} (\bar{y}_{ika|s_2=1} - \bar{y}_{ika|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1})$$

$$Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ika|u=1} - \bar{y}_{ikt|u=1}$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikc|u=1} - \bar{y}_{ikbc|u=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1}$$

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) = \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

Proof. The causal parameter that we are interested in estimating is the average causal effect of the event on the truthful responses to question k of the n^* always-responders:

$$\bar{\tau}_{k|u=1} = \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1})$$

The statistic that we use to estimate this parameter is just

$$\hat{\tau}_{k|u=1} = \frac{1}{n_a} \sum_{i=1}^{n_a} y_{ikao|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikbo|r_b=1}$$

The bias in this estimator is then

$$\begin{aligned}
Bias(\hat{\tau}_{k|u=1}) &= E[\hat{\tau}_{k|u=1}] - \bar{\tau}_{k|u=1} \\
&= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ika|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) \\
&= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ika|r_a=1} - \frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= E\left[\frac{1}{n_a} \sum_{i=1}^{n_a} y_{ika|r_a=1}\right] - E\left[\frac{1}{n_b} \sum_{i=1}^{n_b} y_{ikb|r_b=1}\right] - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= \frac{1}{w_a} \sum_{i=1}^{w_a} y_{ika|w_2=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|r_b=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} (y_{ikt|u=1} - y_{ikc|u=1}) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= \frac{1}{w_a} \sum_{i=1}^{w_a} y_{ika|w_2=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} + \left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1}\right) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= \frac{1}{w_a} \sum_{i=1}^{w_a} y_{ika|w_2=1} - \bar{y}_{ikt|u=1} + \left(\bar{y}_{ikc|u=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1}\right) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \tag{A66}
\end{aligned}$$

We can expand the first term in the above line as follows:

$$\frac{1}{w_a} \sum_{i=1}^{w_a} y_{ika|w_2=1} = \frac{1}{w_a} \sum_{i=1}^{m_a} y_{ika|s_2=1} + \frac{1}{w_a} \sum_{i=1}^{n^*} y_{ika|u=1} \tag{A67}$$

Note that the second term in this expression can be rewritten as:

$$\begin{aligned}
\frac{1}{w_a} \sum_{i=1}^{n^*} y_{ika|u=1} &= \frac{n^*}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} \\
&= \frac{n^* + (w_a - w_a)}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} \\
&= \frac{n^* - w_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} + \frac{w_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} \\
&= \frac{-m_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} + \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ika|u=1} \\
&= -\frac{m_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} + \bar{y}_{ika|u=1} \tag{A68}
\end{aligned}$$

We can then use Lines A67 and A68 to rewrite the overall bias term in Line A66 as

$$Bias(\hat{\tau}_{k|u=1}) = \frac{1}{w_a} \sum_{i=1}^{m_a} y_{ika|s_2=1} - \frac{m_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} + \bar{y}_{ika|u=1} - \bar{y}_{ikt|u=1} + \tag{A69}$$

$$\left(\bar{y}_{ikc|u=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1}\right) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \tag{A70}$$

The second difference in Line A72 can be rewritten as simply the average difference between the n^* always-responders' truthful answers to question k in the world where they completed Survey B in Wave 1 and in the world where they did not. In other words, it is the average causal effect of completing Survey B in Wave 1 on the always-responders' Wave 2 truthful answers. We can think of this possible difference as a potential type of priming bias:

$$Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) = \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ika|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikt|u=1} = \bar{y}_{ika|u=1} - \bar{y}_{ikt|u=1} \quad (\text{A71})$$

Therefore, we can rewrite the overall bias as

$$Bias(\hat{\tau}_{k|u=1}) = \frac{1}{w_a} \sum_{i=1}^{m_a} y_{ika|s_2=1} - \frac{m_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} + Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) + \quad (\text{A72})$$

$$\left(\bar{y}_{ikc|u=1} - \frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} \right) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (\text{A73})$$

We can now turn our attention to the first difference in Line A73. We can begin by noting that

$$\frac{1}{w_b} \sum_{i=1}^{w_b} y_{ikb|w_1=1} = \frac{1}{w_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1} + \frac{1}{w_b} \sum_{i=1}^{n^*} y_{ikb|u=1} \quad (\text{A74})$$

We can break the second term down further as

$$\begin{aligned} \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} &= \frac{n^*}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} \\ &= \frac{n^* + (w_b - w_b)}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} \\ &= \frac{n^* - w_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} + \frac{w_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} \\ &= \frac{-m_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} + \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} \\ &= -\frac{m_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} + \bar{y}_{ikb|u=1} \\ &= \bar{y}_{ikb|u=1} - \frac{m_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} \end{aligned} \quad (\text{A75})$$

Using Lines A74 and A75, we can now rewrite the overall bias term from Lines A72-A73 as

$$\begin{aligned}
Bias(\hat{\tau}_{k|u=1}) &= \frac{1}{w_a} \sum_{i=1}^{m_a} y_{ika|s_2=1} - \frac{m_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} + Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) + \\
&\quad \bar{y}_{ikc|u=1} - \left(\frac{1}{w_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1} + \bar{y}_{ikb|u=1} - \frac{m_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} \right) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\
&= \frac{1}{w_a} \sum_{i=1}^{m_a} y_{ika|s_2=1} - \frac{m_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} + Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) + \\
&\quad (\bar{y}_{ikc|u=1} - \bar{y}_{ikb|u=1}) + \frac{m_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} - \frac{1}{w_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1} + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}
\end{aligned}$$

Similar to what we did in Section 2 of the paper, we can separate the expression $\bar{y}_{ikc|u=1} - \bar{y}_{ikb|u=1}$ into the bias caused by temporal factors and the bias caused by anticipatory factors:

$$\bar{y}_{ikc|u=1} - \bar{y}_{ikb|u=1} = Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1})$$

where

$$\begin{aligned}
Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) &= \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikc|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikbc|u=1} = \bar{y}_{ikc|u=1} - \bar{y}_{ikbc|u=1} \\
Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) &= \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikbc|u=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1}
\end{aligned}$$

As in Section 2, the term y_{ikbc} represents individual i 's Wave 1 truthful response in the counterfactual world where the event “did not happen.” Interpretation of this term depends on the counterfactual that the researcher has in mind.

The overall bias in the estimator can now be written as

$$Bias(\hat{\tau}_{k|u=1}) = \frac{1}{w_a} \sum_{i=1}^{m_a} y_{ika|s_2=1} - \frac{m_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} + Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + \quad (A76)$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \frac{m_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} - \frac{1}{w_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1} + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (A77)$$

We can next note that

$$\begin{aligned}
\frac{1}{w_a} \sum_{i=1}^{m_a} y_{ika|s_2=1} - \frac{m_a}{w_a n^*} \sum_{i=1}^{n^*} y_{ika|u=1} &= \left(\frac{m_a}{w_a} \right) \left(\frac{1}{m_a} \sum_{i=1}^{m_a} y_{ika|s_2=1} - \frac{1}{n^*} \sum_{i=1}^{n^*} y_{ika|u=1} \right) \\
&= \left(\frac{m_a}{w_a} \right) (\bar{y}_{ika|s_2=1} - \bar{y}_{ika|u=1}) \quad (A78)
\end{aligned}$$

Similarly,

$$\begin{aligned} \frac{m_b}{w_b n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} - \frac{1}{w_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1} &= \left(\frac{m_b}{w_b} \right) \left(\frac{1}{n^*} \sum_{i=1}^{n^*} y_{ikb|u=1} - \frac{1}{m_b} \sum_{i=1}^{m_b} y_{ikb|s_1=1} \right) \\ &= \left(\frac{m_b}{w_b} \right) (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1}) \end{aligned}$$

We can now rewrite the overall bias in the estimator from Lines A76-A77 as

$$\begin{aligned} Bias(\hat{\tau}_{k|u=1}) &= \frac{m_a}{w_a} (\bar{y}_{ika|s_2=1} - \bar{y}_{ika|u=1}) + Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + \\ &Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1}) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \\ &= \frac{m_a}{w_a} (\bar{y}_{ika|s_2=1} - \bar{y}_{ika|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1}) + \end{aligned} \quad (\text{A79})$$

$$Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1} \quad (\text{A80})$$

The expression in Line A79 can be labeled

$$Bias_{\mathbf{D}}(\hat{\tau}_{k|u=1}) = \frac{m_a}{w_a} (\bar{y}_{ika|s_2=1} - \bar{y}_{ika|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1})$$

This bias term captures the bias induced by differential dropout rates based on which survey respondents were assigned in Wave 1.

The final difference in Line A80 is just the potential bias caused by differential misreporting between the respondents in Waves 1 and 2. We can label this bias

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) = \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

Therefore, we can now write the overall bias term in condensed form as

$$Bias(\hat{\tau}_{k|u=1}) = Bias_{\mathbf{D}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) + Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1})$$

where

$$Bias_{\mathbf{D}}(\hat{\tau}_{k|u=1}) = \frac{m_a}{w_a} (\bar{y}_{ika|s_2=1} - \bar{y}_{ika|u=1}) + \frac{m_b}{w_b} (\bar{y}_{ikb|u=1} - \bar{y}_{ikb|s_1=1})$$

$$Bias_{\mathbf{P}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ika|u=1} - \bar{y}_{ikt|u=1}$$

$$Bias_{\mathbf{T}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikc|u=1} - \bar{y}_{ikbc|u=1}$$

$$Bias_{\mathbf{A}}(\hat{\tau}_{k|u=1}) = \bar{y}_{ikbc|u=1} - \bar{y}_{ikb|u=1}$$

$$Bias_{\mathbf{M}}(\hat{\tau}_{k|u=1}) = \bar{\epsilon}_{ika|w_2=1} - \bar{\epsilon}_{ikb|w_1=1}$$

□

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